Time domain simulation of the diatonic harmonica

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Abstract

In order to understand the physical behavior of the diatonic harmonica, we present a computer simulation that leads to the inner over-pressure and reeds motion signals. A physical model of the instrument is proposed featuring non-linearities, the two-reed dynamic and a vocal tract. The agreement between numerical simulations and experiment is satisfactory. For each playing mode, we find signals very close to those observed during measurements. These results validate our physical model and give us a good sound synthesis.

1 Introduction

The diatonic harmonica is a wind instrument still poorly studied. More attention have been paid to instruments with striking reed as the clarinet or the saxophone [1], [2], [3] and there are far less works related to instruments with free reeds as the diatonic harmonica or the accordeon[4], [5], [6]. As all musically self-sustained oscillators, the diatonic harmonica is characterized by a noticeable non-linear behavior which gives to the diatonic harmonica sound a very rich harmonic content.

One of the methods commonly used to solve this kind of problem is time-domain simulation. It was first applied to wind instruments such as clarinet and saxophone by Schumacher [7]. Nevertheless, the time-domain model proposed can not be extended to the diatonic harmonica for many reasons. First, it is constituted by two reeds that interact. Moreover, the resonator description is non-linear: therefore, the impedance formalism is not suitable. Consequently, a specific discrete time-domain simulation based on a physical model is proposed. We also explain how to obtain the discrete system similar to the analog one and finally numerical and experimental results are compared.

2 The physical model

The physical model of the diatonic harmonica proposed is a one-dimensional like model which may be used to obtain time simulation and good quality synthesis. It is composed of a mechanical linear oscillator for free reeds and a non-linear part describing the air flow through the instrument and the player taking into account the mean flow.

2.1 Modeling reeds movement

The active element of the diatonic harmonica consists of two reeds said to be free and strong: no obstacle stops the reed motion and reeds vibrate around their first eigen frequency. With measurements using strain gauges, it has been shown that reeds motion is composed of sinusoidal oscillations because of the non-striking nature of the reeds [5]. Considering each reed ($n = 1, 2$) as a one-degree of freedom damped oscillator, the displacement of the reed tip is assumed to be gouverned by :

$$M_n \frac{d^2 h_n}{dt^2} + R_n \frac{dh_n}{dt} + K_n (h_n - h_{n00}) = S_r \Delta p_4 \quad (1)$$

where K_n, M_n and R_n are respectively the equivalent stiffness, the equivalent mass and the equivalent damping coefficient which are calculated from experimental data, h_n the reed opening and h_{n00} the offset position of the reed.

2.2 Modeling the vocal tract

The second step consists in taking into account the vocal tract. Indeed, the oral cavity has to be modelled in order to explain all playing modes (blown, drawn, bends and overnotes) and chromatical playing. In this study, the model of the vocal tract is derived from the one proposed by Fant [8] to approximate vowel production. In fact, Millot [9] linked oral action and production of all playing modes. As a result, our vocal tract is approximated by a combination of four tubes as illustrated in figure 1 with three control parameters in order to define the oral configuration: S_2 is the palatal constriction area, x the position of the center of the palatal constriction and L_4 the length of the lippal and harmonica channel.

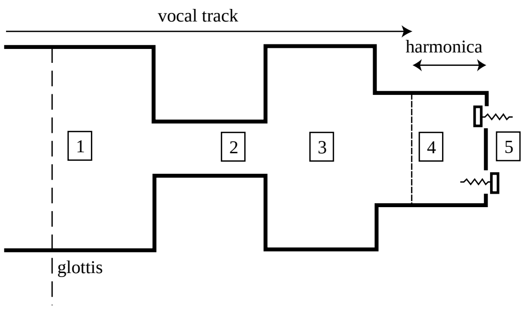

FIG. 1: Model of the vocal tract using a four-cylinder combination: supply area (1), palatal constriction (2), front cavity (3), lippal and harmonica channel (4), outside (5).

The description of the vocal tract uses mass conservation equation for elements (3) and (4) and instationnary and incompressible Bernoulli equations for the palatal constriction (2) and the lippal channel (4). Thus, this description is typically non-linear and we can not linearise these equations as non-linear terms have a significative contribution in the physical phenomenon as it could be shown with our simulations.

3 Description of air flow

The other non-linear part of our model concerns the description of the output air flow. We assumed that, for each reed, we have two contributions for the ouput flow: the flow through the reed u_{tn} and the flow induced by the reed movement u_{an} (or pumped flow).

The volume flow through reed n is given by

$$u_{tn} = \alpha . S_n(h_n) . v_5 \quad , \qquad (2)$$

where α is the *vena contracta* coefficient, v_5 the air flow velocity in element 5 and $S_n(h_n)$ the useful section through the flow escapes from the harmonica.
The expression for the useful section S_n is the following :

$$\begin{aligned} S(h_n) &= B_{rn} . \sqrt{h_{en}^2(L_{rn}) + e^2} + 2 \int_{y/h_{en} \le 0} e \, dy \\ &+ 2 \int_{y/h_{en} > 0} \sqrt{h_{en}^2(y) + e^2} \, dy \ , \qquad (3) \end{aligned}$$

with

- B_{rn} reed width;
- L_{rn} reed length;
- $h_{en}(L_{rn})$ effective height at free-end;
- $h_{en}(y)$ local effective height;
- e_s shallot thickness;
- e_{rn} reed thickness;
- e existing aperture between the reeds and the shallot.

The second contributions to the ouput flow is given by

$$u_{an} = S_r \frac{dh_n}{dt} \ , \qquad (4)$$

where S_r is the effective hydrodynamic area.
Under some assumptions, the model is based upon Bernoulli and mass conservation equations as suggested in [10].
By combination of equations describing the vocal tract and the air flow in the instrument, we obtained two non-linear equations.

Finally, for each case (blow or draw), the effective model is constituted by a set of four equations: two linear differential equations describing reeds motion and two non-linear equations describing the air flow through the instrument.

4 The discrete system

The numerical system corresponding to our problem is found by using signal processing tools. Attention should be paid to the following criteria: accuracy, stability, robustness and rapidity.

4.1 Reeds dynamic in the discrete-time domain

A good discrete approximation for reed motion is obtained using the bilinear transform method as it is suggested in [3]. Our expressions for reeds tip positions are those found by Gazengel.

4.2 Non-linear equations

Concerning differential non-linear equations relative to the air flow, we use the backward Euler method. The non-linear system may be solved for a couple of related parameters $(\Delta p_4, v_2)$.

4.3 Solving the system

An iterative process is developped. For each simulation, the vocal tract configuration, the shape of the excitation signal and the initial conditions are first chosen. Concerning the excitation signal, we have chosen a measured inner pressure signal for a blown note which has been hardly filtered. Next, two non-linear equations in the discrete-time domain are successively solved, defining the volume flow upstream the reeds. We use Newton-Raphson method associated with dichotomy when Newton's process fails. Once we find the corresponding pressure drop Δp_4, the process is repeated with another sample.

As for initial conditions, both pressure and flow are assumed to be equal to zero at the beginning and each reed is at its rest position. Finally, this method gives the playing frequency, reeds motion and inner pressure signals.

5 Comparison between numerical and experimental results

The next step of our study is to compare numerical results with measurements done on the diatonic harmonica.

5.1 Experimental device

Our experimental device is composed by the following parts : an harmonica, two pairs of strain gauges for reeds motion, a differential pressure transducer for the pressure in the instrument and a data acquisition system installed on a computer in order to record the three signals (reeds opening and inner pressure). We notice that experimental and numerical results were obtained with the fourth channel on a G diatonic harmonica.

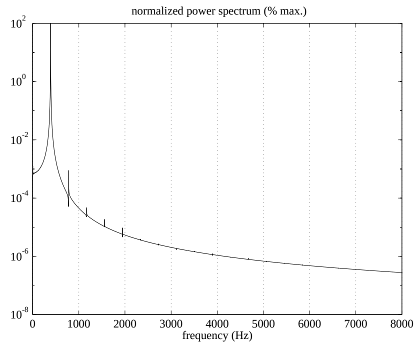

FIG. 2: Normalized power spectrum of the upper reed motion for a normal blow on channel 4.

5.2 Reeds motion

The analysis of reeds motion signals makes clear the fact that reeds motion remains sinusoidal during all playing modes as it is shown on figure 2.

Moreover, as predicted for normal modes, the pitch of the played note is found around the reed eigen frequency and active reed and passive reed are clearly distinguished. Therefore, the agreement between numerical and experimental results concerning reeds motion is correct.

5.3 Inner pressure

Studying the inner pressure signals shows that our physical modelling is satisfactory. Indeed, inner pressure waveforms are very close to those observed during measurements as it is illustrated for a normal blow and an overblow on figures 4, 5. The presence of sharp peaks in the inner pressure signals explain the very rich harmonic content of the sound produced by the diatonic harmonica illustrated on figure 3.

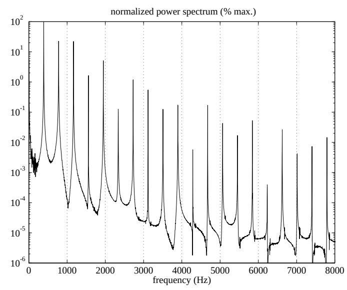

FIG. 3: Normalized power spectrum for the inner over-pressure during a blow on channel 4.

Besides, fine details as beats during final transient are also found and the reeds influence on the inner pressure can also be investigated precisely as it is done in [9].

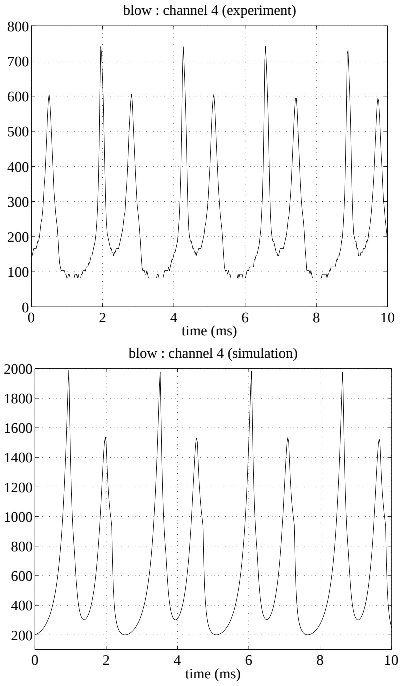

FIG. 4: Comparison of the experimental overpressure Δp_4 for a blown note (channel 4 on a G harmonica, note: G) and the one resulting from the numerical simulation (on the right). The parameters are : $\Delta p_{1\ max} = 470$ Pa, $x = 4.5$ cm.

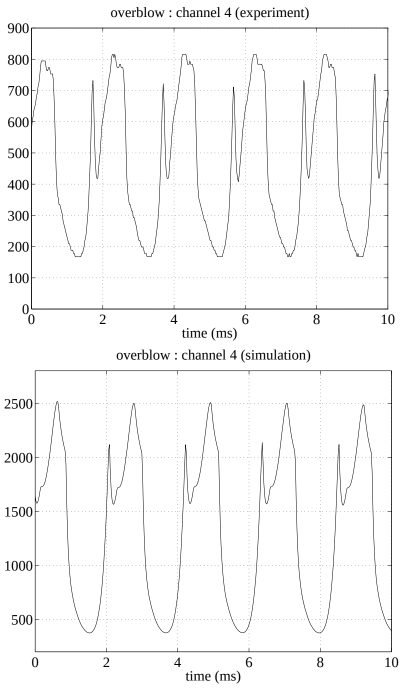

FIG. 5: Comparison of the experimental overpressure Δp_4 for a overblown note (channel 4 on a G harmonica) and the one resulting from the numerical simulation (on the right). The parameters are : $\Delta p_{1\ max} = 900$ Pa, $x = 11$ cm.

6 Conclusion

A one-dimensional physical model of the diatonic harmonica was described. Thanks to discrete-time simulations, we succeed in solving the non-linear system and we access to the reeds motion and the inner pressure signals. Looking at the presented results, we can conclude that our physical model is coherent. We also succeed in performing chromatical playing on a diatonic harmonica using different vocal tract configurations on the same channel. This opens the way to further investigations on the diatonic harmonica in order to understand more precisely the parameters influence.

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